$f(x)=x\ln x$ $g(x)=a\mathrm{e}^x\left(a\in\ \right)$

$y=f(x)$ $x=1$ $y=g(x)$ a

$G(x)=f(x)-g(x)$

a $a\mathrm{e}^2$ 2 $G(x)$ 0

$a=\mathrm{e}^{-2}$ a $\left(0,\frac{1}{\mathrm{e}}\right)$

$F(x)=\frac{G(x)}{x}=\ln x-\frac{a\mathrm{e}^x}{x}$ $F'(x)=\frac{x-a(x-1)\mathrm{e}^x}{x^2}$

0 x a $\frac{2}{\mathrm{e}^2}$ $F'(x)$ 0 $F(x)$ $(0,1)$

$F(x)$ $F(1)=-a\mathrm{e}$ 0

x $F'(x)=-\frac{a(x-1)}{x^2}\left[\mathrm{e}^x-\frac{x}{a(x-1)}\right]$

$H(x)=\mathrm{e}^x-\frac{x}{a(x-1)}$ $H'(x)=\mathrm{e}^x+\frac{1}{a(x-1)^2}$ 0

a $\frac{2}{\mathrm{e}^2}$ $H(2)=\frac{a\mathrm{e}^2-2}{a}$ 0 $m\in(1,2)$ $\frac{m}{a(m-1)}$ e^2

1 m $\frac{a\mathrm{e}^2}{a\mathrm{e}^2-1}$ $H(m)=\mathrm{e}^m-\frac{m}{a(m-1)}$ $\mathrm{e}^2-\mathrm{e}^2=0$ $H(m)\cdot H(2)$ 0

$H(x)$ $x_0\in(1,2)$ $F(x)$ $x_0\in(1,2)$

$H(x_0)=0$ $\mathrm{e}^{x_0}=\frac{x_0}{a(x_0-1)}$ $F(x_0)=\ln x_0-\frac{1}{x_0-1}$ $x_0\in(1,2)$

$F'(x_0)=\frac{1}{x_0}+\frac{1}{(x_0-1)^2}$ 0 $F(x_0)$ $(1,2)$

$F(x_0)$ $F(2)=\ln 2-\frac{a\mathrm{e}^2}{2}$ $\ln 2-1$ 0

$ae^2 \quad 2$ $\quad \frac{G(x)}{x} \quad 0$ $\quad G(x) \quad 0$

$G(x)$

$F(x)=\frac{G(x)}{x}=\ln x-\frac{ae^x}{x}$

$F'(x)$

$0 \quad x$ $\quad F'(x) \quad 0$ $\quad F(x) \quad 0$ $\quad x$ $\quad F(x)$

$F'(x)=-\frac{a(x-1)}{x^2}\left[e^x-\frac{x}{a(x-1)}\right]$ $\quad H(x)=e^x-\frac{x}{a(x-1)}$ $\quad H(x)$

$(1,2)$ $\quad x_0$ $\quad F(x_0)$ $\quad F(x_0) \quad 0$

$G(x) \quad 0 \Leftarrow G(x)_{\max} \quad 0 \Leftarrow F(x)_{\max} \quad 0$

x

$ae^2 \quad 2$ $\quad a \quad \frac{2}{e^2}$ $\quad G(x)=x\ln x-ae^x<0$ $\quad a>\frac{x\ln x}{e^x}$

(i) $\quad 0<x<1$ $\quad \frac{x\ln x}{e^x}<0$

(ii) $\quad x \quad 1$ $\quad h(x)=\frac{x\ln x}{e^x}$ $\quad h'(x)=\frac{\ln x+1-x\ln x}{e^x}$

$u(x)=\ln x+1-x\ln x,(x \quad 1)$ $\quad u'(x)=\frac{1}{x}-\ln x-1$

$u'(x)$ $\quad [1,+\infty)$ $\quad u'(x) \quad u'(1)=0$ $\quad u(x) \quad [1,+\infty)$

$u(2)=1-\ln 2>0$ $\quad u(3)=1-2\ln 3<0$

$\exists x_0\in(2,3)$ $\quad h'(x_0)=0$ $\quad \ln x_0+1-x_0\ln x_0=0$

$1<x<x_0$ $\quad h'(x_0)>0$ $\quad h(x)$ $\quad x>x_0$ $\quad h'(x_0)<0$ $\quad h(x)$

$h(x) \quad h(x_0)=\frac{x_0\ln x_0}{e^{x_0}}=\frac{\ln x_0+1}{e^{x_0}}$

$<\frac{x_0}{e^{x_0}}<\frac{2}{e^2}$ $\quad \ln x_0<x_0-1,$ $\quad y=\frac{x}{e^x}$ $\quad 2,3$

(i) (ii) $\quad a>\frac{x\ln x}{e^x}$

$a>\frac{x\ln x}{e^x}$ $\quad \frac{2}{e^2}$ $\quad \left(\frac{x\ln x}{e^x}\right)_{\min}$

$h(x)=\frac{x\ln x}{e^x}$ $\quad h'(x)=0$ $\quad x_0$ $\quad$ m a

$x\ln x \quad ae^{x} \qquad ae^{2} \quad 2 \qquad x\ln x \quad 2e^{x-2}$

$\frac{\ln x}{x} \quad \frac{2e^{x-2}}{x^{2}} \qquad \varphi(x)=\frac{\ln x}{x} \qquad \varphi(x) \qquad (0,e)$

$(e,+\infty) \qquad \varphi(x) \quad \varphi(e)=\frac{1}{e}$

$m(x)=\frac{2e^{x-2}}{x^{2}} \qquad m'(x)=\frac{2e^{x-2}(x-2)}{x^{3}}$

$m(x) \qquad (0,2) \qquad (2,+\infty)$

$m(x) \quad m(2)=\frac{1}{2} \quad \frac{1}{e} \quad \varphi(x) \qquad \frac{\ln x}{x} \quad \frac{2e^{x-2}}{x^{2}}$

$ae^{2} \quad 2 \qquad x\ln x \quad 2e^{x-2}$

$\frac{\ln x}{x} \quad \frac{2e^{x-2}}{x^{2}} \qquad \left(\frac{\ln x}{x}\right)_{max} \quad \left(\frac{2e^{x-2}}{x^{2}}\right)_{min}$

$\ln x \quad \frac{2e^{x-2}}{x} \quad 0 \qquad \ln x \quad e^{x}$

$x\ln x \quad 2e^{x-2} \qquad \frac{2e^{x-2}}{x} \quad \ln x$

$\varphi(x)=\frac{2e^{x-2}}{x} \quad x=2 \qquad y=\frac{1}{2}x \qquad \frac{2e^{x-2}}{x} \quad \frac{1}{2}x$

$x=2 \qquad \frac{1}{2}x \quad \ln x \qquad \frac{2e^{x-2}}{x} \quad \ln x$

$\varphi(x)=\frac{2e^{x-2}}{x} \qquad g(x)=a(2\ln x+1)$

$f(x)=ae^{x}-\ln x-1$

$x=2 \quad f(x) \qquad a \qquad f(x)$

$a \quad \frac{1}{e} \qquad f(x) \quad 0$

$f(x)=e^{2x}-a\ln x$

$f\left(x\right)$ $f'\left(x\right)$

$a>0$ $f\left(x\right)$ $2a+a\ln\dfrac{2}{a}$

$f\left(x\right)=ae^x\ln x+\dfrac{be^{x-1}}{x}$ $y=f(x)$

$\left(1,f(1)\right)$ $y=e(x-1)+2$

a,b $f(x)>1$

$f\left(x\right)=\dfrac{2\ln x+2}{e^x}$

$f\left(x\right)$

$x>0$ $f'\left(x\right)\ln\left(x+1\right)<\dfrac{2}{e^x}+\dfrac{2}{e^{x+2}}$

$f\left(x\right)$ $\left(0,1\right)$ $\left(1,+\infty\right)$

$f'\left(x\right)\ln\left(x+1\right)<\dfrac{2}{e^x}+\dfrac{2}{e^{x+2}}$ $\left(1-x-x\ln x\right)\ln\left(x+1\right)<\left(1+\dfrac{1}{e^2}\right)x$

$g\left(x\right)=1-x-x\ln x$ $g'\left(x\right)=-1-\left(\ln x+1\right)=-2-\ln x$

$0<x<\dfrac{1}{e^2}$ $g'\left(x\right)>0$ $x>\dfrac{1}{e^2}$ $g'\left(x\right)<0$

$g\left(x\right)$ $\left(0,\dfrac{1}{e^2}\right)$ $\left(\dfrac{1}{e^2},+\infty\right)$ $g\left(x\right)$ $1-\dfrac{1}{e^2}+\dfrac{2}{e^2}=1+\dfrac{1}{e^2}$

$1-x-x\ln x$ $1+\dfrac{1}{e^2}$ $\ln\left(x+1\right)<x$ $x=1$

$0<\ln\left(x+1\right)<x$ $\left(1-x-x\ln x\right)\ln\left(x+1\right)<\left(1+\dfrac{1}{e^2}\right)x$

$x>0$ $f'\left(x\right)\ln\left(x+1\right)<\dfrac{2}{e^x}+\dfrac{2}{e^{x+2}}$

$\ln x$ e^x

$\ln x$ $x-1\Leftrightarrow e^x$ $x+1$

x $e^{2x}-a\ln x$ $\dfrac{1}{2}a$ a

$2e^{2x}$ $a\left(2\ln x+1\right)$ $f\left(x\right)=2e^{2x}$ $g\left(x\right)=a\left(2\ln x+1\right)$

a 0 a 0

a 0 t

$\begin{cases} f(t)=g(t) \\ f'(t)=g'(t) \end{cases}$ $\begin{cases} 2e^{2t}=a(2\ln t+1) \\ 4e^{2t}=\dfrac{2a}{t} \end{cases}$ $\begin{cases} t=1 \\ a=2e^2 \end{cases}$

a $\left[0,2e^2\right]$

e^{2x} $a\left(\ln x+\frac{1}{2}\right)$ e^{2x} $a\ln\left(\sqrt{e}x\right)$ $f(x)=e^{2x}$ $a\ln\left(\sqrt{e}x\right)$ 0

a 0 a 0

a 0 e^{2x} $\frac{a}{2}\ln\left(\sqrt{e}x\right)^2$ $\frac{a}{2}$ $\frac{e^{2x}}{\ln\left(ex^2\right)}$ e^{2x} $\left(e^x\right)^2$ $(ex)^2$

$\ln\left(ex^2\right)$ $\frac{1}{e}\cdot ex^2=x^2$ $x=1$ $\left(\frac{e^{2x}}{\ln\left(ex^2\right)}\right)_{\min}=\frac{(ex)^2}{x^2}=e^2$

$\frac{a}{2}$ e^2 0 a $2e^2$ a $\left[0,2e^2\right]$

$f(x)=2e^{2x}$ _ 6 /ë $g(x)=a(2\ln x+1)$

$\left(\frac{e^{2x}}{\ldots}\right)_{\min}$

[redacted]